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BT-2/M-20 **32001**

MATHEMATICS-II

Paper–MATH-102E

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

UNIT–I

1. (a) For what value of k, the equations

$$\left.\begin{array}{llll} x & y & z & 1 \\ 2x & y & 4z & k \\ 4x & y & 10z & k^2 \end{array}\right\}$$ have a solution ? Solve them

completely in each case.

(b) Find Eigen value and Eigen vectors for

$$A \begin{bmatrix} 6 & 2 & 2 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{bmatrix}.$$

2. (a) State the following :

(i) Rank of a Matrix. 2

(ii) Unitary Matrix. 2

(iii) Skew-Hermitian Matrix. 3

(iv) Cayley-Hamilton Theorem. 3

(b) Verify Cayley Hamilton theorem for the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$. Hence compute A^{-1}.

UNIT–II

3. (a) Solve the differential equations $\frac{dy}{dx} \quad \frac{y}{x \quad \sqrt{xy}}$.

(b) An e.m.f. E sin pt is applied at t = 0 to a circuit containing a Capacitance C and Inductance L. The current x satisfies the equation :

$$L\frac{dx}{dt} \quad \frac{1}{C}\int x\, dt \quad E \sin pt.$$

If $p^2 \quad \frac{1}{LC}$ and initially the current x and the charge q are zero, show that the current at time t is $\left(\frac{Et}{2L}\right)\sin pt$,

where $x \quad \frac{dq}{dt}$.

4. (a) Find the solution of $\frac{d^2y}{dx^2} \quad 3\frac{dy}{dx} \quad 2y \quad xe^x \sin x$.

(b) Solve the Legendre's linear equation,

$$(1 \quad x)^2 \frac{d^2y}{dx^2} \quad (1 \quad x)\frac{dy}{dx} \quad y \quad 4\cos(\log(1 \quad x)).$$

UNIT–III

5. (a) Show that

$$L^{\ 1}\left[\frac{1}{s}\cos\frac{1}{s}\right] \quad 1 \quad \frac{t^2}{(2!)^2} \quad \frac{t^4}{(4!)^2} \quad \frac{t^6}{(6!)^2} \quad .....$$

(b) Find the Laplace transforms of $\frac{1 \quad \cos t}{t^2}$.

6. (a) Using transformation method, solve the differential equation

$$t\frac{d^2y}{dt^2} \quad 2\frac{dy}{dt} \quad ty \quad \sin t, \text{ when } y(0) = 1.$$

(b) Find the Laplace transform of a periodic function

$f(t) \quad k\frac{t}{T}$ for $0 < t < T$, representing saw tooth wave.

UNIT–IV

7. (a) Solve the equation,

$$(z^2 - 2yz - y^2)p + (xy + xz)q = (xy - zx).$$

(b) Using Charpit's method, solve the equation

$$pxy + pq + qy = yz.$$

8. (a) Obtain the complete solution of the equation

$$yp = 2xy + \log q.$$

(b) Derive the general solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$.